Exercise 20

For the following exercises, sketch the parametric equations by eliminating the parameter. Indicate any asymptotes of the graph.

$$x = 4\sec\theta, \quad y = 3\tan\theta$$

Solution

Solve each of these equations for $\sec\theta$ and $\tan\theta$.

$$\frac{x}{4} = \sec\theta, \quad \frac{y}{3} = \tan\theta$$

Square both sides of each equation and then subtract the respective sides.

$$\left(\frac{x}{4}\right)^2 - \left(\frac{y}{3}\right)^2 = \sec^2\theta - \tan^2\theta$$

$$\frac{x^2}{16} - \frac{y^2}{9} = (\tan^2\theta + 1) - \tan^2\theta$$

$$= 1.$$

This is the graph of a hyperbola opening along the x-axis. Below is a plot of the parametric equations for $0 \le t \le 2\pi$.